PhD Admission Test (Dept. of CSE) IIT Guwahati

Time: 10:30 - 12:30 (2 Hrs.) **Date: June 3, 2024**

Question & Answer Booklet: (13 Questions, 10 Marks each)

Name:		**App. No.:**	
Signature:		**Category:** (Gen/OBC/ SC/ST/PwD)	

Instructions

1. This question set has three sections namely **Mathematical Foundations**; **Programming, Data Structure and Algorithms**; and **Computer Systems**.
2. You can attempt any question from any section. However, you have to write which **TEN** answers we should evaluate. You **Must** mention those questions numbers below in the provided box.
3. Ensure that there are 4 printed sheets.
4. Answers should be written **only** within the space provided. Answers written outside the provided space **will not** be evaluated.
5. **No clarifications** on questions will be entertained. State your assumptions, if any.
6. Ask for separate sheets for **Rough Work**. **Do Not** do any rough work in this booklet.
7. Write your **name** and **application number** on the extra sheets also, if any taken for rough work, and return the sheets after the exam.
8. There is **NO negative marks** for incorrect answer of any question.

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Answer any TEN questions and put their question numbers

Answered question #											Total
Marks (Office use only)											

Section 1: Mathematical Foundations

Q1. (a) Solve the following recurrence relation. [5]

$$T(0) = 0;$$
$$T(n) = 2T(n-1) + 1, \text{ for } n > 0.$$

(b) Solve the following recurrence relation. [5]

$$T(1) = 1;$$
$$T(n) = 9T\left(\frac{n}{3}\right) + n, \text{ for } n > 1.$$

Q2. (a) Use the generating function method to find a formula for u_n defined as follows [5]

$$u_0 = 1;$$
$$u_1 = 1;$$
$$u_{n+2} - 4u_{n+1} + 4u_n = 0, \text{ where } n \geq 0.$$

(b) How many ways can we arrange the twenty-six letters of the English alphabet so that no two vowels occur together? [5]

Q3. (a) A long-distance running race had 15 participants, and among them were Amy and Bob. How many outcomes are possible if we know that Amy finished ahead of Bob? [5]

(b) We toss an unbiased coin five times. What is the probability that we get two Heads? [5]

Q4. (a) If we toss a coin five times, what is probability that the results are alternating? [5]

(b) If the functions f and g are bijections, and the function $g \circ f$ is well-defined, then prove that the inverse of $g \circ f$ is $f^{-1} \circ g^{-1}$. [5]

Section 2: Programming, Data Structure and Algorithms

Q5. There are four people who want to cross a bridge at night. They have only one torch and the bridge is only wide enough to allow two people to cross at a time. It takes each person a different amount of time to cross the bridge (given in minutes): A takes 1 minute, B takes 2 minutes, C takes 5 minutes, and D takes 10 minutes. When two people cross the bridge together, they must move at the slower person's pace. Carrying a torch is a must while crossing the bridge. What is the minimum amount of time needed for all four people to cross the bridge? [10]

Q6. Consider the weighted undirected simple complete graph K_n with n vertices and $\frac{n(n-1)}{2}$ edges. Vertices are indexed from 1 to n. The Weight of each edge connecting vertices i and j is $i+j$. We are building the minimum spanning tree for this graph using the Kruskal's Algorithm. We are using disjoint set forest to represent components. We are using union by size heuristic while merging the components. Suppose that we have inserted m edges in the minimum spanning tree ($0 < m < n$). How many components will be there? For each component, list the vertices. [10]

Q7. Using decision tree model depict working of insertion sort on any input of size three. [10]

Q8. Consider the following algorithm for sorting a given set of integers. Is the algorithms correct? If your answer is no then, provide an input on which the algorithm will fail to sort the input correctly. If your answer is yes, prove the correctness of the algorithm. [10]

```
/* a[0] to a[n-1] is the array to sort */
void sortAlgo (int[] a, int n)
{
    int i, j, k, h, v;
    int[] cols = {1391376, 463792, 198768, 86961, 33936,
                  13776, 4592, 1968, 861, 336, 112, 48,
                   21, 7, 3, 1}
    for (k=0; k<16; k++)
    {
        h=cols[k];
        for (i=h; i<n; i++)
        {
            v=a[i];
            j=i;
            while (j>=h && a[j-h]>v)
            {
                a[j]=a[j-h];
                j=j-h;
            }
            a[j]=v;
        }
    }
}
```

Section 3: Computer Systems

Q9. (a) Explain the difference between data hazard and control hazards in an instruction pipeline with the help of a few instruction sequences. [5]

(b) A 4-bit register R stores number in 2'complement form. What is the smallest and largest number that can be stored in R. [2]

(c) A logic block called 'X' is placed between processor and memory. An address given by a processor will pass through X to select a unique memory location. Which one of the following component is most suitable for X? Explain.
(i) Encoder (ii) Multiplexer (iii) Decoder (iv) Demultiplexer [3]

Q10. (a) A processor uses 4 KB direct mapped cache memory. The cache uses a block size of 16 bytes. What is the tag, set index and offset of the 16-bit physical address if we assume 1 word is 1 byte. [6]

(b) What are the addressing modes of the following instructions. [4]
SUB R1, R2, R3 // Subtract contents of R2 from R3 and store result in R1.
ADD R4, #50 // Add contents of R4 with value 50 and store in R4 itself.

Q11. (a) Consider a main memory with three page frames and it uses least recently used (LRU) page replacement algorithm. Assume that all the page frames are initially empty. Find the number of page faults for the page reference string 2, 4, 1, 4, 3, 0, 1, 3. [6]

(b) Match the following. [4]

Set A	Set B
Bankers' algorithm	Translation lookaside buffer (TLB)
Compaction	Deadlock Avoidance
Semaphores	External Fragmentation
Page Table	Process Synchronization

Q12. (a) Name the layer(s) in the OSI reference model the following devices operate. (i) hub (ii) repeater (iii) bridge (iv) router [4]

(b) Explain the difference between TCP and UDP. [6]

Q13. (a) Consider three processes P_1, P_2, and P_3 with CPU bursts 8, 4, and 6 cycles, respectively, scheduled to run on a uniprocessor system. Assume all processes arrived at time $t = 0$. Compute the average turnaround time for these processes under (i) shortest job first scheduling (ii) round robin scheduling with a time quantum of 3 cycles. [8]

(b) In a process state diagram which of the following transitions are not possible? (i) ready to running (ii) waiting to running (iii) ready to waiting (iv) running to waiting [2]